

Mathematical Reminiscence

A Tribute to Winfried Bruns

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- There were almost no journals from the West
- Major sources for information: Mathematical Reviews and Zentralblatt
- If I understood the subject of a paper, I sent a postcard to the author asking for an offprint. Winfried was among them
- There were special postcards for this purpose

Back side of a postcard for offprints

Sehr geehrte Frau Kollegin Sehr geehrter Herr Kollege	Многоуважаемый коллега	Dear Sir	Monsieur et cher confrère
Ich wäre dankbar für die Übersendung eines Sonderdruckes Ihrer Arbeit:	Будьте настолько любезны переслать мне Ваш труд:	I should greatly appreciate a reprint of your article:	Je vous serais très obligé de m'envoyer un tirage à part de votre article intitulé:
Mit kollegialen Grüßen	за что заранее благодарю	Sincerely yours	Je vous en remercie d'avance et vous prie de croire, Monsieur, à mes sentiments les plus distingués
Unterschrift			

0195 Ag 307 III/21/16 85 N 3378 G 8626

A famous paper

V. Tny

Reprinted from JOURNAL OF ALGEBRA
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Vol. 39, No. 2, April 1976
Printed in Belgium

“Jede” endliche freie Auflösung ist freie Auflösung eines von drei Elementen erzeugten Ideals

WINFRIED BRUNS

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Communicated by D. Buchsbaum

Received December 13, 1974

EINLEITUNG

Burch und Kohn haben in [4] bzw. [7] folgendes bewiesen: R sei ein kommutativer noetherscher Ring und n eine natürliche Zahl, die homologische Dimension eines torsionslosen R -Moduls ist; dann existiert ein von höchstens drei Elementen erzeugtes Ideal in R , dessen homologische Dimension gleich n ist. Buchsbaum und Eisenbud [3, p. 135, Conjecture, vgl. auch Theorem 7.2] vermuten sogar, “jede” endliche freie Auflösung sei freie Auflösung eines solchen Ideals. Wir wollen diese Vermutung im Rahmen des folgenden allgemeineren Resultats beweisen:

$$0 \rightarrow F_n \xrightarrow{f_n} F_{n-1} \rightarrow \cdots \rightarrow F_{m+1} \xrightarrow{f_{m+1}} F_m \rightarrow M \rightarrow 0$$

sei eine projektive Auflösung von M , F_m ein freier R -Modul und M ein m -torsionsloser R -Modul (oder äquivalent: ein m -ter Syzygienmodul), der einen Rang besitzt, ferner sei $r := \text{rang Bild } f_{m+1}$; dann existieren Homomorphismen

$$c: F_m \rightarrow R^{r+m}, \quad f_m: R^{r+m} \rightarrow R^{2m-1}, \quad f_j: R^{2j+1} \rightarrow R^{2j-1}, \quad j = 1, \dots, m-1$$

derart, daß mit $f'_{m+1} := c \circ f_{m+1}$ die Sequenz

$$\begin{array}{ccccccc} 0 \rightarrow F_n & \xrightarrow{f_n} & F_{n-1} & \rightarrow & \cdots & \rightarrow & F_{m+1} \xrightarrow{f_{m+1}} F_m \xrightarrow{f'_{m+1}} R^{r+m} \\ & & & & & & \downarrow f_m \\ & & & & & & R^{2m-1} \rightarrow \cdots \rightarrow R^5 \xrightarrow{f_2} R^3 \xrightarrow{f_1} R \end{array}$$

exakt ist (Satz 3). Für $m = 2$ (und freie Moduln F_k , $k = 2, \dots, n$) ist dies die Vermutung von Buchsbaum und Eisenbud.

Independent elements

- I returned to Vietnam in 1978 and were first inspired by the following papers:
- G. Valla, Remarks on generalized analytic independence. Math. Proc. Cambridge Phil. Soc. 85 (1979)
- W. Bruns, On the number of elements independent with respect to an ideal, J. London Math. Soc. 22 (1980)
- My first citation to Winfried:
- N.V. Trung, Maximum number of independent elements and dimension of prime divisors in completions of local rings. J. Algebra 93 (1985)

First encounter

- I was invited by Valla to Genova for 4 months in 1985
- Valla drove me to the Oberwolfach conference
Kommutative Algebra and Algebraische Geometrie (uninvited participation)
- That was the first time I met so many great names in mathematics

Oberwolfach 1985?



A preprint sent to Hanoi that got my interest

Additions to the Theory of Algebras with Straightening Law

WINFRIED BRUNS

In this article we want to supplement the theory of algebras with straightening law, ASLs for short, by two additions. The first addition concerns the arithmetical rank of ideals generated by an ideal of the poset underlying the ASL. (The arithmetical rank is the least number of elements generating an ideal up to radical.) It turns out that there is a general upper bound only depending on the combinatorial data of the poset. In particular we discuss ideals generated by monomials and show that the ideas leading to the general bound can be used to derive sharper results in this special case. On the other hand there exists a class of ASLs, called symmetric, in which the general bound is always precise. This class includes the homogeneous coordinate rings of Grassmannians, and perhaps the most prominent result in this context is the determination of the least number of equations defining a Schubert subvariety of a Grassmannian.

As a second addition we introduce the notion of a module M with straightening law over an ASL A . Such a module has a partially ordered set of generators, a basis of standard elements each of which is a product of a standard monomial and a generator, and finally the multiplication $A \times M \rightarrow M$ satisfies a straightening law similar to the straightening law in A . We discuss some examples, among them the powers of certain ideals generated by poset ideals and the generic modules. Remarkable facts: (i) the first syzygy of a module with straightening law has itself a straightening law, and (ii) the existence of natural filtrations which, for example, lead to a lower bound on the depth of a module with straightening law. In the last part we introduce a natural strengthening of the axioms which under special conditions leads to a straightening law on the symmetric algebra of the module. The most interesting examples to which we will apply this result are the generic modules.

The theory of ASLs has been developed in [Ei] and [DEP.2]; the treatment in [BV.1] also satisfies our needs. For the readers convenience we have collected the definition, results relevant for us, and some significant examples in the first section. Matsumura's book [Ma] may serve as a source for the commutative ring theory needed in this article.

1. ALGEBRAS WITH STRAIGHTENING LAWS

An algebra with straightening law is defined over a ring B of coefficients. In order to avoid problems of secondary importance in the following sections we will assume throughout that B is a noetherian ring.

DEFINITION. Let A be a B -algebra and $\Pi \subset A$ a finite subset with partial order $\leq$. A is an algebra with straightening law on Π (over B) if the following conditions are satisfied:

(ASL-0) $A = \bigoplus_{i \geq 0} A_i$ is a graded B -algebra such that $A_0 = B$, Π consists of homogeneous elements of positive degree and generates A as a B -algebra.

(ASL-1) The products $\xi_1 \cdots \xi_m$, $m \geq 0$, $\xi_1 \leq \cdots \leq \xi_m$ are a free basis of A as a B -module.

Partially supported by a DFG travel grant

Hodge algebras

- W. Bruns, Additions to the theory of algebras with straightening law, Commutative algebra (Berkeley, CA, 1987), Math. Sci. Res. Inst. Publ. 15, Springer, 1989.
- N.V. Trung, On the presentation of Hodge algebras and the existence of Hodge algebra structures, Comm. Algebra 19 (1991)

ACKNOWLEDGEMENT. The author is grateful to W. Bruns who read the first draft of this paper and gave many useful suggestions.

First joint paper

- First 2 papers of Winfried on determinantal rings:
- W. Bruns, Die Divisorenklassengruppe der Restklassenringe von Polynomringen nach Determinantenidealen, Rev. Roum. Math. Pures Appl. 20, (1975)
- W. Bruns, The canonical module of a determinantal ring, Commutative Algebra: Durham, London Math. Soc. Lect. Notes 72, (1981)
- These papers had inspired the following papers:
- A. Simis and N.V. Trung, The divisor class group of ordinary and symbolic blow-ups. Math. Z. 198 (1988).
- W. Bruns, A. Simis, N.V. Trung, Blow-up of straightening-closed ideals in ordinal Hodge algebras. Trans. Amer. Math. Soc. 326 (1991)

Bruns and Vetter: Determinantal rings



A special citation

- The first citations of Winfried to my works appeared in the book, especially
- N.V. Trung, Principal systems of ideals, Acta Math. Vietnamica 6 (1981)
- We exchanged this journal with a few places outside of Vietnam at that time
- How he came to know that paper?
- I still recommend my colleagues at home to read this book if they want to know about determinantal ideals

ICTP 1992

- The first school and conference on Commutative Algebra at ICTP, Sept. 14-25, 1992
- Organizers: A. Simis, N.V. Trung, G. Valla
- There were 4 lecturers for the school: W. Bruns, J. Herzog, G. Lyubeznik, B. Ulrich
- I had to browse Math. Reviews and Zentralblatt looking for possible participants from developing countries
- Some of them told me much later that it was an important for them because of the connections with experts from the west
- Guess: What is the title of Winfried lecture?



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SMR.639/2

WORKSHOP ON COMMUTATIVE ALGEBRA

(14 - 25 September 1992)

Determinantal rings

W. Bruns
Universität Osnabrück
Department of Mathematics
Driverstrasse 22
2848 Vechta
Germany

These are preliminary lecture notes, intended only for distribution to participants

ICTP 1994

- School and Workshop on Commutative Algebra and its Relation to Combinatorics and Computer Algebra, May 16–26, 1994
- There were 5 lecturers for the school: W. Bruns, J. Herzog, B. Sturmfels, B. Ulrich, N.V. Trung
- A more difficult question: What did Winfried lecture about?



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SMR.761/4

**Workshop on Commutative Algebra
and its Relation to
Combinatorics and Computer Algebra**

(16 - 27 May 1994)

**Algebraic and Combinatorial
Reciprocity Laws**

W. Bruns
FB Naturwissenschaften Mathematik
Universität Osnabrück
Standort Vechta
49377 Vechta
Germany

These are preliminary lecture notes, intended only for distribution to participants

The only photos of me and Winfried: Trieste 2010



Hanoi 1996

- International Conference on Algebraic Geometry, Commutative Algebra, and Computation, August 19–23, 1996
- Organizers: D. Eisenbud, N.V. Trung, W. Vogel
- Participants: Aberbach, Brodmann, Bruns, Esnault, Foxby, Hashimoto, Herzog, Hibi, Iarrobino, Miyazaki, Kawamata, Sharp, Sturmfels, Srinivas, Verma, Viehweg, Watanabe, Zarzuela

In a “dust restaurant” 1996



Hanoi 1996 (we were jung)



Hanoi Conference 1996



Second joint paper

- In 1995 I came to Osnabruck for a talk during a longer stay with Herzog in Essen
- We (me, Joseph, Winfried) discussed about the Koszul property of polytopal rings and came up with the idea of triangulations:
- W. Bruns, J. Gubeladze, N.V. Trung, Normal polytopes, triangulations, and Koszul algebras, J. Reine Angew. Math. 485 (1997)
- Some problems of this paper are still open
- This is the most cited paper of Winfried

Third joint paper

- In 2000 me and Winfried, we were invited to participate in the Colloquium on Semigroups in Szeged, Hungary, July-21, 2000.
- The organizers wanted to know about results on affine semigroups:
- W. Bruns, J. Gubeladze, N.V. Trung, Problems and algorithms for affine semigroups, Semigroup Forum 64 (2002)
- This is the most cited paper of this issue (excluding self-citations)

Bruns and Herzog: Cohen-Macaulay rings (2nd edition)



Bruns and Gubeladze: Polytopes, rings, and K-theory



Ha Long Bay 2017

- International Conference on Commutative Algebra and its Interaction to Combinatorics, Discrete Geometry and Singularity Theory, Ha Long Bay, September 11 to 15, 2017
- Participants: Achilles, Brodmann, Bruns, Chardin, Elias, Herzog, Kreuzer, Kwak, Manaresi, Miro-Roig, Miyazaki, Nagel, Nakamura, Polini, Roemer, Schenzel, Terai, Ulrich, Verma, Yassemi, Zarzuela
- It was not easy to convince Winfried to travel so faraway

Front of the conference villa Looking for Winfried: first for Herzog



Ha Long Bay 2017



Ha Long Bay 2017 (on the shoulder of two giants)



Cat Ba 2017



Cat Ba 2017



Mud bath 2017: Winfried was happy with the trip



Recent citation to Winfried's work

- W. Bruns, A. Conca, C. Raicu, M. Varbaro, Determinants, Gröbner bases and cohomology, Springer Monographs in Mathematics, 2022
- He forgot to send a copy to me (that was always the case for the first three books)
- L.T. Hoa, H.D. Nguyen, N.V. Trung, Saturation degree of powers of graded ideals, Preprint 2026

You are always in our memories

