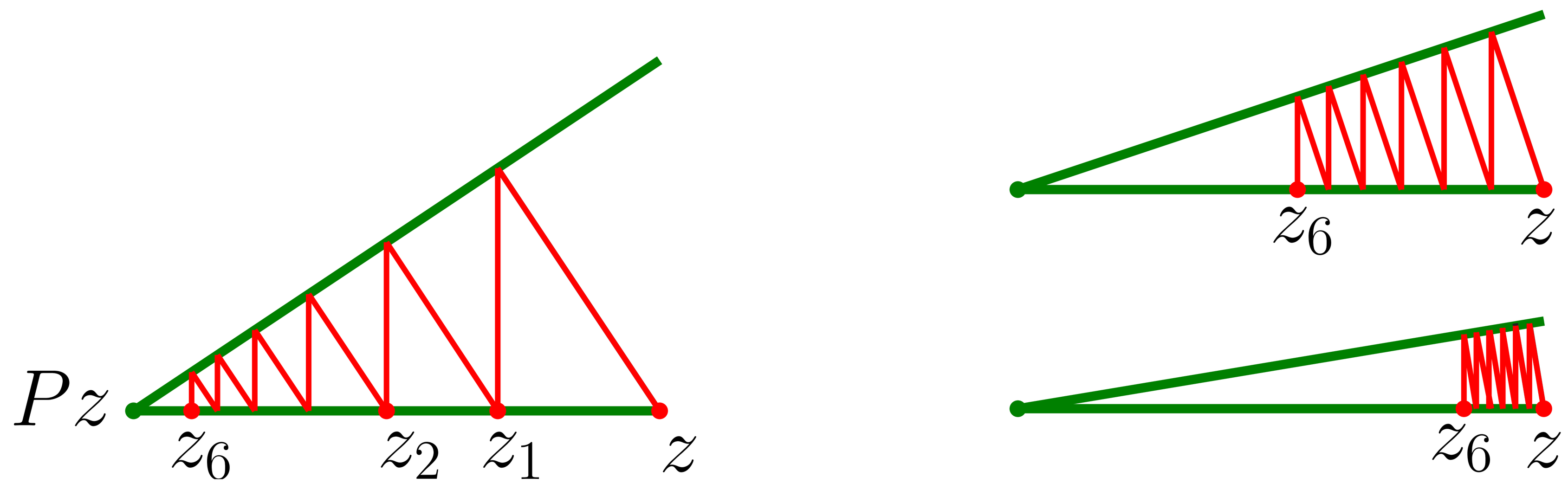
universität innsbruck
Department of Mathematics



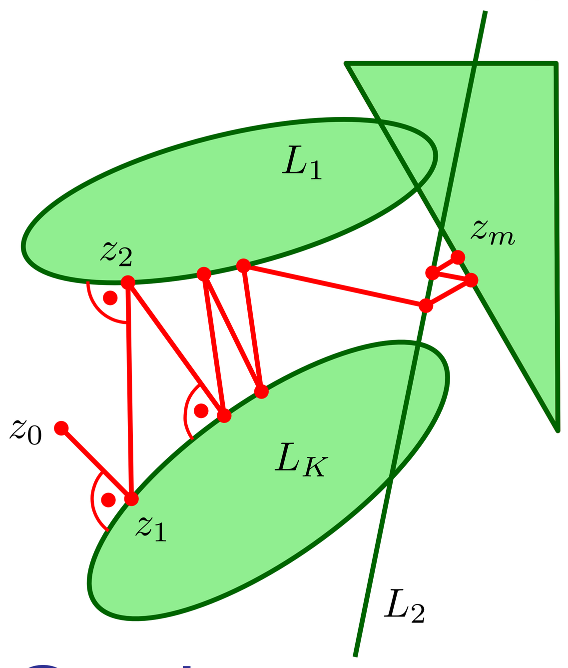
Projections and Approximation

Interplay of alternating projections and greedy approximation

Let H be Hilbert space. Given finitely many closed convex sets in H, iterating nearest point projections onto them can find a point in the intersection of the sets. Given a spanning subset D of H, the greedy algorithm can approximate a given point by linear combinations of elements of D. There is a duality between the two schemes.

Iterates of Projections

$K \in \mathbb{N}$ fixed, *e.g.* $K = 5$
$L_1, L_2, \dots, L_K \subset H$ closed subspaces
$k_1, k_2, \dots \in \{1, 2, \dots, K\}$

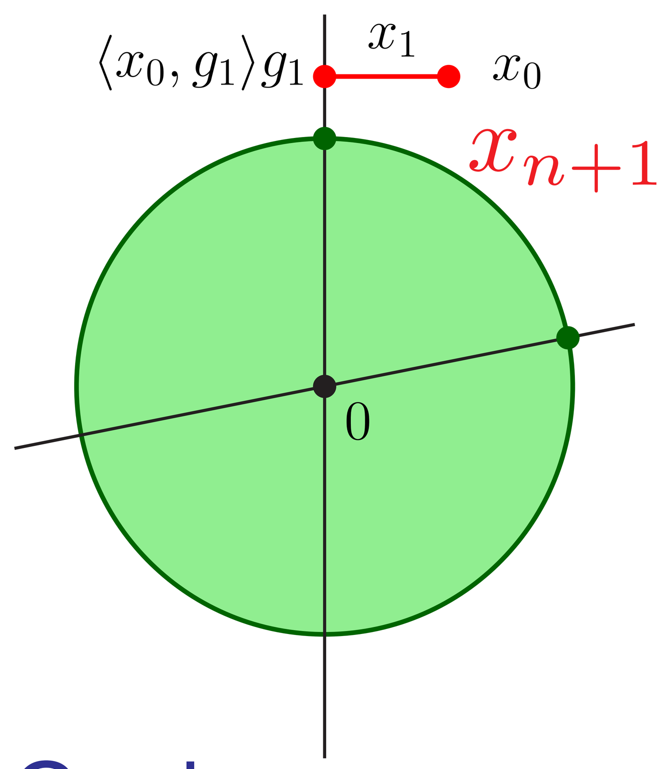


$z_n = P_{k_n} z_{n-1}$
sequence of projections

Goal:
find the projection Pz_0 onto $\bigcap_{k=1}^K L_k$;
i.e. show $z_n \to 0$ if $\bigcap_{k=1}^K L_k = \{0\}$.

Greedy Algorithm

D spanning subset of the sphere of H
let $g_{n+1} \in D$ maximize $|\langle x_n, g \rangle|$



$x_{n+1} = x_n - \langle x_n, g_{n+1} \rangle g_{n+1}$
linear combinations of elements of D

Goal:
approximate x_0 by linear combinations of elements of D; *i.e.* show $x_n \to 0$.

INTERPLAY

• Greedy algorithm converges, hence remotest projections converge as well.
Use $D = (L_1^\perp \cup \dots \cup L_K^\perp) \cap S(H)$.
• Alternating projections converge. They converge fast iff $Y = L_1^\perp + \dots + L_K^\perp$ is closed.
• Greedy algorithm converges fast iff D does not fit in an arbitrarily thin plank.

[Badea, Borodin, Deutsch, Grivaux, Hundal, Jones, Kopecká, V. Müller, ...]

Eva Kopecká
Department of Mathematics
Universität Innsbruck

uibk.ac.at/mathematik/functionalanalysis/