PARIS
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UNIVERSITÄT
SALZBURG

Conference on Convex Geometry and Geometric Probability

September 25 – 29, 2023

Salzburg University

Foto: Luigi Caputo

Program

Monday, 25.09.2023

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	Coffee break
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11:10 – 11:35	**Marius Tiba** and **Peter van Hintum** *Sharp stability for the Brunn-Minkowski inequality for arbitrary sets (p. 16)*
11:35 – 12:00	**Ning Zhang** *Uniqueness of convex bodies by non-central sections in the plane (p. 19)*
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Locations

The talks are taking place in the Faculty of Natural Sciences (Naturwissenschaftliche Fakultät) of Salzburg University in lecture hall 402 (Blauer Hörsaal). The address is

Hellbrunner Straße 34
5020 Salzburg, Austria
Phone (Department of Mathematics): +43 662 8044 5300.

Poster Session

The poster session on Monday September 25^{th}, 2023 takes place in the foyer in front of lecture hall 402.

Excursion

For participants who registered for the excursion on Wednesday September 27^{th}, 2023 to the Lake Wolfgang the pick-up point is at 13:45 in front of the Faculty of Natural Sciences. We are going by bus to St. Gilgen and then by boat to St. Wolfgang.

Concert and Reception

The concert on Thursday September 28^{th}, 2023 begins at 19:00 and takes place in the historical library auditorium (Bibliotheksaula) of Salzburg University in the center of Salzburg's old town. The address of the library auditorium is

Hofstallgasse 2–4
5020 Salzburg, Austria.

Due to a limited number of seats in the library auditorium the concert can only be attended by registered participants.

After the concert there will be a reception in the foyer in front of the library auditorium.

Abstracts

A flag representation for an n-dimensional convex body

Rafik Aramyan, Institute of Mathematics of NAS RA

Abstract. The cosine transforms plays a fundamental role in convex geometry and a number of related areas. It is known that (see [2]) *the support function H of a sufficiently smooth origin symmetric convex body $\boldsymbol{B}$ in $\mathbf{R}^n$ has the cosine representation with an even signed measure defined on the unit $n-1$ dimensional sphere $\mathbf{S}^{n-1}$.*
Our aim is to generalize the cosine transform and find a new representation for an n-dimensional convex body in terms of surface curvature functions of the body.
By definition: a flag is an ordered pair of orthogonal unit vectors in $\mathbf{R}^n$ and we use the representation (Ω, Φ) for the flag, where $\Omega \in \mathbf{S}^{n-1}$ is the spatial direction of the first vector and Φ is the direction of the second (perpendicular to Ω). The function

$$\rho(\Omega, \Phi, \xi) = \frac{\langle \xi, \Omega \rangle^2}{\sin^{n-1}(\widehat{\xi, \Phi})}$$

defined for $\Omega, \xi \in \mathrm{S}^{n-1}$, $\Phi \in \mathrm{S}_\Omega$ (the great $n-2$ dimensional sphere with pole at Ω) $(\xi \neq \Phi)$ is called *the flag density* function (for $\xi = \Phi$ we assume that $\rho = 0$).

Theorem 1. *The support function of an origin symmetric 2-smooth convex body $\mathbf{B} \subset \mathbf{R}^n$ has the following representation. For $\xi \in \mathbf{S}^{n-1}$ (see [1])*

$$H(\xi) = \frac{(n-1)!!}{2\,\sigma_{n-2}} \int_{\mathrm{S}^{n-1}} \int_{\mathrm{S}_\Omega} \frac{\langle \xi, \Omega \rangle^2}{\sin^{n-1}(\widehat{\xi, \Phi})} \frac{\sqrt{K(\Phi)}}{(k(\Phi, \Omega))^{\frac{n+1}{2}}} \, \lambda_{n-2}(d\Phi)\, \lambda_{n-1}(d\Omega),$$

here $K(\Phi)$ is the Gauss-Kronecker curvature at the point with the outer normal Φ on $\partial\mathbf{B}$ and $k(\Phi, \Omega)$ is the normal curvature at the same point in direction $\Omega \in \mathrm{S}_\Phi$. λ_k is the spherical Lebesgue measure on $\mathbf{S}^k$ and σ_k is the total measure of $\mathbf{S}^k$.

As a consequence of the representation we propose a sufficient condition for an origin symmetric n-dimensional convex body to be a zonoid. The condition written in terms of surface curvature functions of the convex body and has a local equatorial description.

References: [1] *R. H. Aramyan*: A Flag Representation for a n-Dimensional Convex Body, The Journal of Geometric Analysis, Volume 29, Issue 3, pp 2998 – 3009, 2019. [2] *W. Weil, R. Schneider*: Zonoids and related Topics, in: P. Gruber, J. Wills (Eds), Convexity and its Applications, Birkhauser, Basel, pp. 296 – 317, 1983.

Thursday, 28.09.2023, 11:35 – 12:00

Universal sets and sequences of lines and k-flats in $\mathbb{R}^d$

Imre Bárány, Alfréd Rényi Institute of Mathematics

Abstract. According to the famous Erdös-Szekeres theorem among any $N = 4^n$ points in the plane (in general position) there are n that form the vertices of a convex n-gon. In other words, the appearance of n points in convex position is **universal** (among sufficiently many points in $\mathbb{R}^2$). Are there universal sets (or sequences) of lines and of k-flats in $\mathbb{R}^d$? If so, what are they? In this talk I will give partial answers to these questions. Joint work with Gil Kalai and Attila Por.

Wednesday, 27.09.2023, 9:30 – 10:15

Mean distance in convex polyhedra, mean tetrahedron volume and related problems

Dominik Beck, Charles University in Prague

Abstract. I. Mean distance between two randomly selected points chosen uniformly from the interior of a given convex polyhedron K was known in the exact form only for K being a cube (the Robbins constant). However, a modification of the Crofton Reduction Technique always turns the problem into finite series of solvable double integrals. This way, we managed to derive the exact mean distance for all other regular polyhedra (tetrahedron, octahedron, dodecahedron, icosahedron). As the procedure can be done for any polyhedron, the mean distance is always expressible in the exact form.

II. The mean tetrahedron volume problem asks to find the expected volume of a convex hull of four points (which form a tetrahedron almost surely) selected uniformly and independently from the interior of K. In 90's (Buchta and Reitzner) and 00's (Zinani), Efron's formula was used to deduce the mean volume in the case of K being a tetrahedron and a cube, respectively. Shortly after Christmas 2020, using the same Efron's formula with more optimal parametrisation and heavily relying on computer algebra system Mathematica, we extended the result for K being a regular octahedron. The exact value turned out to be

$$\frac{19297\pi^2}{3843840} - \frac{6619}{184320} = 0.013637411276524175460212315329967798293238477874952877 86...$$

In subsequent months of 2021, we also found the exact mean tetrahedron volume in six other polyhedra: triangular prism, square pyramid, rhombic dodecahedron, cuboctahedron, triakis tetrahedron, truncated octahedron. In our presentation, we shortly outline the general method how we obtained these results and also briefly discuss how we could proceed in higher dimensions as the Efron's formula possesses a simple generalisation there.

Tuesday, 26.09.2023, 16:15 – 16:40

Floating Bodies and Polarity in Non-Euclidean Geometries

Florian Besau, Technische Universität Wien

Abstract. Meyer & Werner showed that Lutwak's p-affine surface area in d-dimensional Euclidean space arises as the volume derivative of the floating body of convex body conjugated by polarity for $p = -d/(d+2)$. We establish an extension of this relation in the spherical and hyperbolic space. Our results hold in spaces of constant curvature, and we also show that the Euclidean result of Meyer & Werner can be obtained by a limiting process as the space curvature tends to zero.
Based on joint work with E. Werner.

Tuesday, 26.09.2023, 11:35 – 12:00

Typical and extremal results for random polytopes in smooth convex bodies

Pierre Calka, CNRS – Université de Rouen Normandie

Abstract. We consider the random polytope generated by n independent and uniformly distributed points in a smooth convex body. We are interested in studying the typical and extremal distributions of several functionals of the facets of the random polytope, which are the distance to the boundary, the volume and the diameter. Each of those functionals is coupled with the relative position of the facet, which is given by a boundary point of the underlying smooth convex body. We exhibit in particular explicit limit distributions for the typical distributions and prove similar explicit convergences of measures associated to the extreme value regime. This leads us in particular to getting limiting extreme value distributions for the distance to the boundary and for the volume. Those results reflect the radial and longitudinal fluctuations of the random polytope which are reminiscent of those of a large class of random growth interfaces.

Thursday, 28.09.2023, 9:30 – 10:15

Brunn-Minkowski type inequalities for variational functionals in Gauss space

Andrea Colesanti, Università degli Studi di Firenze

Abstract. There are several examples of Brunn-Minkowski type inequalities verified by variational functionals. Significant examples of such functionals are: the electrostatic capacity, the torsional rigidity and the first Dirichlet eigenvalue of the Laplace operator. We investigated the possibility to extend these inequalities to the Gauss space. In this talk I will present some results in this direction, concerning the Gaussian first Dirichlet eigenvalue of the Laplace operator. These results were obtained in collaboration with E. Francini, G. Livshyts and P. Salani.

Friday, 29.09.2023, 9:30 – 10:15

Strengthened inequalities for the mean width and the ℓ-norm

Ferenc Fodor, University of Szeged

Abstract. Barthe (1998) proved that the regular simplex maximizes the mean width of general convex bodies whose John ellipsoid is the Euclidean unit ball. Equivalently, the regular simplex is the maximizer for the ℓ-norm of general convex bodies whose Löwner ellipsoid is the Euclidean unit ball. Schmuckenschläger (1999) proved the corresponding reverse statements, and he also established the origin-symmetric cases where the extremizers are the cube and the regular cross-polytope. In this talk we discuss stronger stability versions of these inequalities. The arguments are based on the strengthening of Barthe's measure transportation proofs of the geometric Brascamp–Lieb and reverse Brascamp–Lieb inequalities for special classes of functions. (Joint work with Károly J. Böröczky and Daniel Hug.)

Monday, 25.09.2023, 10:45 – 11:10

X-rays in Convex Geometry

Richard Gardner, Western Washington University

Abstract. In 1917, the great Austrian mathematician Johann Radon published a paper that initiated tomography as a mathematical subject. Nowadays mathematical X-rays are of several types, such as parallel or point, and continuous or discrete. The talk focuses on continuous parallel X-rays and attempts to survey their role in convex geometry. For example, they underlie the radial pth mean bodies introduced in 1998 by the speaker and Gaoyong Zhang. Though the latter have attracted considerable attention recently, mathematical X-rays have in general been somewhat neglected. With this in mind, a selection of appealing open problems will be presented in the hope of reviving interest.

Monday, 25.09.2023, 15:00 – 15:45

How does the sectional Poisson-Voronoi tessellation look like?

Anna Gusakova, Westfälische Wilhelms-Universität Münster

Abstract. A Poisson-Voronoi tessellation is one of the classical models in stochastic geometry constructed as a Voronoi diagram of a homogeneous Poisson point process in $\mathbb{R}^d$. In this talk we will consider the ℓ-dimensional tessellations appearing as an intersection of d-dimensional Poisson-Voronoi tessellation with an ℓ-dimensional, $1 \leq \ell \leq d-1$, hyperplane. We will give their precise description and study their main characteristics, which will also allow to shed a new light on the properties of classical Poisson-Voronoi tessellation.
Based on the joint work with Christoph Thäle and Zakhar Kabluchko.

Thursday, 28.09.2023, 15:00 – 15:45

Equality cases in the Alexandrov–Fenchel inequality and area measures of convex bodies

Daniel Hug, Karlruher Institut für Technologie

Abstract. Mixed volumes in n-dimensional Euclidean space are functionals of n-tuples of convex bodies $K, L, C_1, \dots, C_{n-2}$. The Alexandrov–Fenchel inequalities (AFI) are fundamental inequalities between mixed volumes of convex bodies, which cover as very special cases many important inequalities between basic geometric functionals. Up to now a complete characterization of the equality cases in the Alexandrov–Fenchel equality is missing. Major recent progress is due to Yair Shenfeld and Ramon von Handel, in particular they resolve the problem in the cases where K, L are general convex bodies and $C_1, \dots, C_{n-2}$ are polytopes, zonoids or smooth bodies. We introduce a new class of convex bodies, which includes polytopes and zonoids as special examples and extend their study of the extremals of the AFI to this class. A related study concerns the determination of the support of mixed area measures, which provide a local counterpart of the mixed volumes. (Based on joint work with Paul A. Reichert.)

Tuesday, 26.09.2023, 15:00 – 15:45

Coarse approximations in convex analysis

Grigory Ivanov, Institute of Science and Technology Austria

Abstract. Sparse approximations of positive definite matrices and polytopes have attracted attention due to several recent breakthroughs. The idea is to approximate a given object with a specified small error by a relatively simple structured object of a certain type. For example, one can approximately represent a positive definite matrix of order d by a matrix of rank $c(\varepsilon)d$, where ε is the error tolerance. However, the case of "coarse approximation", where the simplest structure capable of approximating an object within a larger error remains largely unexplored. In particular, for a convex polytope in $\mathbb{R}^d$ defined as the intersection of half-spaces, the minimum number of half-spaces needed to approximate the polytope within an error of d remains unknown. In our talk, we will discuss coarse approximation in convex analysis, shedding light on this and similar problems.

Tuesday, 26.09.2023, 16:40 – 17:05

Palm Theory on Unimodular Random Measured Metric Spaces

Ali Khezeli, INRIA Paris

Abstract. In this work, we define the notion of unimodular random measured metric spaces as a common generalization of various other notions. This includes the discrete cases like unimodular graphs and stationary point processes, as well as the non-discrete cases like stationary random measures and the continuum metric spaces arising as scaling limits of graphs. We provide various examples and prove many general results; e.g., on weak limits, re-rooting invariance, random walks, ergodic decomposition, amenability and balancing transport kernels. In addition, we generalize the Palm theory to point processes

and random measures on a given unimodular space. This is useful for Palm calculations and also for reducing some problems to the discrete cases.

Wednesday, 27.09.2023, 11:10 – 11:35

Monge-Ampère operators and valuations

Jonas Knörr, Technische Universität Wien

Abstract. Monge-Ampère-type operators play an important role in many problems in analysis and geometry. Many of these operators can naturally be considered as measure-valued valuations on spaces of convex functions, and consequently, they have found a number of applications in the construction of invariant valuations on convex bodies and convex functions. In this talk, I will present a characterization of a certain class of measure-valued valuations and different descriptions of these functionals in terms of mixed Monge-Ampère operators and differential forms.

Monday, 25.09.2023, 16:40 – 17:05

Inequalities and Counterexamples for Functional Intrinsic Volumes

Fabian Mußnig, Technische Universität Wien

Abstract. Intrinsic volumes on convex bodies satisfy a range of important inequalities such as the isoperimetric inequality or the Brunn-Minkowski inequality. Recently, functional intrinsic volumes on convex functions were introduced and characterized in a Hadwiger-type theorem. It is only natural to ask whether the new functional versions of the intrinsic volumes also satisfy similar inequalities. We will not only show counterexamples, but also explain with the help of results by Colesanti, Hug and Saorín Gómez why certain inequalities fail. Last but not least, we will also present positive results.
Joint work with Jacopo Ulivelli.

Monday, 25.09.2023, 16:15 – 16:40

Higher rank antipodality

Márton Naszódi, Alfréd Rényi Institute of Mathematics

Abstract. Motivated by general probability theory, we say that the set X in $\mathbb{R}^d$ is *antipodal of rank* k, if for any $k+1$ elements $q_1, \dots q_{k+1} \in X$, there is an affine map from conv$X$ to the k-dimensional simplex Δ_k that maps $q_1, \dots q_{k+1}$ onto the $k+1$ vertices of Δ_k. For $k = 1$, it coincides with the well-studied notion of (pairwise) antipodality introduced by Klee. We consider the following natural generalization of Klee's problem on antipodal sets: What is the maximum size of an antipodal set of rank k in $\mathbb{R}^d$? We present a geometric characterization of antipodal sets of rank k and adapting the argument of Danzer and Grünbaum originally developed for the $k = 1$ case, we prove an upper bound which is exponential in the dimension. We point out that this problem can be connected to a

classical question in computer science on finding perfect hashes, and it provides a lower bound on the maximum size, which is also exponential in the dimension. Joint work with Zsombor Szilágyi and Mihály Weiner.

Tuesday, 26.09.2023, 11:10 – 11:35

Fixed Points of Minkowski Valuations

Oscar Ortega-Moreno, Technische Universität Wien

Abstract. In this talk, we show that for any sufficiently regular even Minkowski valuation Φ which is homogeneous and intertwines rigid motions, there exists a neighborhood of the unit ball, where balls are the only solutions to the fixed-point problem $\Phi^2 K = \alpha K$. This significantly generalizes results by Ivaki for projection bodies and suggests, via the Lutwak–Schneider class reduction technique, a new approach to Petty's conjectured projection inequality.

Thursday, 28.09.2023, 16:15 – 16:40

Pseudo-cones, copolarity, and Minkowski type problems

Rolf Schneider, Albert-Ludwigs-Universität Freiburg

Abstract. Recently, several authors have pointed out that pseudo-cones, together with their copolarity, can be considered as a counterpart to the set of convex bodies containing the origin in the interior, with their ordinary polarity. A pseudo-cone in $\mathbb{R}^n$ is a nonempty closed convex set K, not containing the origin, such that together with $x \in K$ also $\lambda x \in K$ for all $\lambda \geq 1$. The copolar set of a pseudo-cone K is defined by $K^* = \{x \in \mathbb{R}^n : \langle x, y\rangle \leq -1 \text{ for all } y \in K\}$ and is a pseudo-cone satisfying $K^{**} = K$. The first part of the talk gives more details about copolarity. The major part deals with the surface area measure of pseudo-cones. It can be constructed as for convex bodies, but is only defined on an open subset of a hemisphere, and can be infinite. A Minkowski type problem asks for necessary and sufficient conditions on a measure to be the surface area measure of a pseudo-cone with given recession cone. No complete answer is known, but we consider several partial solutions.

Friday, 29.09.2023, 11:35 – 12:20

Boolean models in hyperbolic space

Matthias Schulte, Technische Universität Hamburg

Abstract. The union of the grains of a stationary Poisson process of compact convex sets in Euclidean space is called Boolean model and is a classical topic of stochastic geometry. In this talk, Boolean models in hyperbolic space are considered. They are obtained as unions of the grains of isometry invariant Poisson processes on the compact convex subsets of the hyperbolic space. Geometric functionals such as volume of the intersection of

the Boolean model with a ball as observation window are studied. For increasing radius of the ball, asymptotic formulas for expectations, variances and covariances are shown and univariate and multivariate central limit theorems are derived. Compared to the the Euclidean case, some new phenomena can be observed.
This talk is based on joint work with Daniel Hug and Günter Last (both Karlsruhe).

Friday, 29.09.2023, 11:10 – 11:35

Spherical Centroid Bodies

Franz Schuster, Technische Universität Wien

Abstract. In this talk, the spherical centroid body of a centrally-symmetric convex body in the Euclidean unit sphere is introduced. Two alternative definitions – one geometric, the other probabilistic in nature – are given and shown to lead to the same objects. The geometric approach is then used to establish a number of basic properties of spherical centroid bodies, while the probabilistic approach inspires the proof of a spherical analogue of the classical polar Busemann–Petty centroid inequality.
(Joint work with F. Besau, T. Hack, and P. Pivovarov.)

Thursday, 28.09.2023, 16:40 – 17:05

How Coxeter and Escher meet Poisson

Christoph Thäle, Ruhr-Universität Bochum

Abstract. In this talk we discuss Poisson hyperplane tessellations in hyperbolic space. More generally, we consider Poisson process of λ-geodesic hyperplanes. It is demonstrated how the transition from the Euclidean to the hyperbolic world leads to new and unexpected phenomena, which are related to the effect of observing unbounded cells or unconventional limit distributions.

Wednesday, 27.09.2023, 11:35 – 12:20

Sharp stability for the Brunn-Minkowski inequality for arbitrary sets

Marius Tiba and Peter van Hintum, University of Oxford

Abstract. The Brunn-Minkowski inequality states that for (open) sets A and B in $\mathbb{R}^d$, we have $|A+B|^{1/d} \geq |A|^{1/d} + |B|^{1/d}$. Equality holds if and only if A and B are convex and homothetic sets in $\mathbb{R}^d$. In this talk, we present a sharp stability result for the Brunn-Minkowski inequality, concluding a long line of research on this problem. We show that if we are close to equality in the Brunn-Minkowski inequality, then A and B are close to being homothetic and convex, establishing the exact dependency between the three notions of closeness. This is based on joint work with Alessio Figalli, Peter van Hintum, and Marius Tiba.

Monday, 25.09.2023, 11:10 – 11:35

Intrinsic volumes of polyconvex excursion sets of Poisson shot noise processes

Vanessa Trapp, Technische Universität Hamburg

Abstract. Excursion sets of Poisson shot noise processes are a prominent class of random sets. We consider a specific class of Poisson shot noise processes whose excursion sets in a compact convex observation window are almost surely polyconvex. In this talk we are interested in the behaviour of the intrinsic volumes of these excursion sets for growing observation windows. In particular, we study the asymptotics of the expectation and the variance and derive a central limit theorem.

Friday, 29.09.2023, 10:45 – 11:10

A new connection between the volume product and regularization of heat flow

Hiroshi Tsuji, Osaka University

Abstract. We discuss the Blaschke–Santaló inequality and Mahler's conjecture from convex geometry regarding the volume product. A key ingredient in our talk is the smoothing property of Ornstein–Uhlenbeck semigroup called hypercontractivity. Our first result is to construct a new connection between the Blaschke–Santaló inequality and improved Borell's reverse hypercontractivity. Moreover, we give a new proof of the Blaschke–Santaló inequality via an improvement of Borell's reverse hypercontractivity. Our second result is to give forward hypercontractivity with a special exponent, which is naturally motivated from our first result to solve Mahler's conjecture. This result yields a new lower bound of the volume product for convex bodies whose boundaries are well curved.

Thursday, 28.09.2023, 10:45 – 11:10

Stereological determination of particle size distributions for similar convex bodies

Thomas van der Jagt, Delft University of Technology

Abstract. In the classical Wicksell problem spheres are randomly positioned in space. The problem is to determine the size distribution of the spheres using the circular section profiles observed in a planar section. We study a generalization of this problem. Consider an opaque medium which contains 3D particles. All particles are convex bodies of the same shape, but they vary in size. The particles are randomly positioned and oriented within the medium and cannot be observed directly. Taking a planar section of the medium we obtain a sample of observed 2D section profile areas of the intersected particles. We obtain an identifiability result stating that the profile area distribution uniquely determines the 3D size distribution. Moreover, via the Mellin-Stieltjes transform we derive an inversion formula for this particle size distribution. Of particular importance in this problem is the distribution associated with the area of an Isotropic Uniformly Random (IUR) section of a fixed convex body. We show that for various classes of convex bodies this distribution is absolutely continuous with respect to Lebesgue measure. Finally, we

introduce a likelihood-based non-parametric estimator for the particle size distribution function.
Joint work with Geurt Jongbloed and Martina Vittorietti.

References: [1] T. van der Jagt, G. Jongbloed, and M. Vittorietti, "Existence and approximation of densities of chord length- and cross section area distributions", *arXiv preprint arXiv:2305.02864*, 2023. [2] T. van der Jagt, G. Jongbloed, and M. Vittorietti, "Stereological determination of particle size distributions for similar convex bodies", *arXiv preprint arXiv:2305.02856*, 2023.

Wednesday, 27.09.2023, 10:45 – 11:10

Approximation of convex bodies in Hausdorff distance by random polytopes

Elisabeth Werner, Case Western Reserve University

Abstract. While there is extensive literature on approximation, deterministic as well as random, of general convex bodies in the symmetric difference metric, or other metrics coming from intrinsic volumes, very little is known for corresponding random results in the Hausdorff distance.

For a polygon Q in the plane, the convex hull of n points chosen at random on the boundary of Q gives a random polygon Q_n. We determine the exact limiting behavior of the expected Hausdorff distance between Q and a random polygon Q_n as the number n of points chosen on the boundary of Q goes to infinity.

Based on joint work with J. Prochno, C. Schuett and M. Sonnleitner.

Tuesday, 26.09.2023, 9:30 – 10:15

The Minkowski Problem in Integral Geometry

Dongmeng Xi, Shanghai University

Abstract. We introduced a new family of translation invariant geometric measures arising from Integral Geometry of convex bodies. These measures are related to a family of new Monge-Ampère type operators converging to a σ_k operator. The Minkowski problems for these new measures are proposed and attacked. This is joint work with Erwin Lutwak, Deane Yang, and Gaoyong Zhang.

Thursday, 28.09.2023, 11:10 – 11:35

Radial fluctuations of random convex hulls

Joseph Yukich, Lehigh University

Abstract. Denote by K_n the convex hull of n i.i.d. random variables uniformly distributed in a smooth d-dimensional convex set K. It is shown that the re-scaled radial fluctuations of the boundary of K_n asymptotically converge to an explicit limit distribution as $n \to 1$, with Tracy-Widom like tails in $d = 2$. When K is the unit disc, the

radial fluctuations satisfy process level convergence. We introduce a dual space-time two parameter growth process, which for $t = 1$ coincides with the support function of K_n, which displays $1 : 2 : 3$ scaling, and which converges to a two-parameter limit process given by the Hopf-Lax formula.
This talk is based on joint work with Pierre Calka (Université de Rouen).

Tuesday, 26.09.2023, 10:45 – 11:10

Uniqueness of convex bodies by non-central sections in the plane

Ning Zhang, Huazhong University of Science and Technology

Abstract. In this talk, we will present a recent work about the uniqueness of 2-dimensional convex bodies determined by their noncentral chords, which follows from the symmetry of convex curves with respect to circular arc.

Monday, 25.09.2023, 11:35 – 12:00

Borsuk's Partition Problem, Hadwiger's Covering Conjecture, and the Boltyanski-Gohberg Conjecture

Chuanming Zong, Tianjin University

Abstract. In 1933, K. Borsuk proposed the following problem: *Can every bounded set in the n-dimensional Euclidean space be divided into $n+1$ subsets of smaller diameters?* In 1957, H. Hadwiger made the following conjecture: *Every n-dimensional convex body K can be covered by 2^n translates of its interior $int(K)$.* In 1965, V. G. Boltyanski and I. T. Gohberg made the following conjecture: *Every bounded set in an n-dimensional normed space can be divided into 2^n subsets of smaller diameters.* These problems are closely related. Up to now, all of them are far away from being completely solved.

In this talk, we will introduce a computer approach to these problems. In particular, we will show an asymptotic solution to the Boltyanski-Gohberg conjecture.

Monday, 25.09.2023, 9:30 – 10:15

List of Participants

Rafik Aramyan (National Academy of Sciences of Armenia)
Sotirios Armeniakos (Technische Universität Wien)
Imre Bárány (Alfréd Rényi Institute of Mathematics)
Dominik Beck (Charles University in Prague)
Florian Besau (Technische Universität Wien)
Krishnendu Bhowmick (Österreichische Akademie der Wissenschaften)
René Brandenberg (Technische Universität München)
Leo Brauner (Technische Universität Wien)
Victor-Emmanuel Brunel (ENSAE/CREST Paris)
Christian Buchta (Universität Salzburg)
Xiaxing Cai (Technische Universität Wien)
Pierre Calka (CNRS – Université de Rouen Normandie)
Andrea Colesanti (Università degli Studi di Firenze)
Susanna Dann (Universidad de los Andes)
Emil Dare (Aarhus University)
Lianne de Jonge (Universität Osnabrück)
Simon Ellmeyer (Technische Universität Wien)
Ferenc Fodor (University of Szeged)
Ansgar Freyer (Technische Universität Wien)
Richard Gardner (Western Washington University)
Javier Martín Goñi (Universität Passau)
Florian Grundbacher (Technische Universität München)
Anna Gusakova (Westfälische Wilhelms-Universität Münster)
Bernhard Hafer (Universität Osnabrück)
Daniel Hug (Karlruher Institut für Technologie)
Mathias in Wolde-Lübke (Westfälische Wilhelms-Universität Münster)
Grigory Ivanov (Institute of Science and Technology Austria)
Michael Juhos (Universität Passau)
Zakhar Kabluchko (Westfälische Wilhelms-Universität Münster)
Ali Khezeli (INRIA Paris)
Jonas Knörr (Technische Universität Wien)
Tomasz Kobos (Jagiellonian University Cracow)
Eva Kopecka (Universität Innsbruck)
Dylan Langharst (Kent State University)
Minh Tri Le (Technische Universität Wien)
Monika Ludwig (Technische Universität Wien)
Fernanda Helen Moreira Baeta (Technische Universität Wien)

Mohamed Abdeldjalil Mouamine (Technische Universität Wien)
Jaouad Mourtada (ENSAE/CREST Paris)
Fabian Mußnig (Technische Universität Wien)
Shohei Nakamura (Osaka University)
Márton Naszódi (Alfréd Rényi Institute of Mathematics)
Oscar Ortega-Moreno (Technische Universität Wien)
Davide Ravasini (Universität Innsbruck)
Matthias Reitzner (Universität Osnabrück)
Christian Richter (Friedrich-Schiller-Universität Jena)
Mia Runge (Technische Universität München)
Adam Sagmeister (Eötvös Loránd University)
Rolf Schneider (Albert-Ludwigs-Universität Freiburg)
Matthias Schulte (Technische Universität Hamburg)
Franz Schuster (Technische Universität Wien)
Bettina Sereinig (Universität Salzburg)
Gurvinder Singh (Akash Global University India)
Mathias Sonnleitner (Universität Passau)
David Steigenberger (Westfälische Wilhelms-Universität Münster)
Anna Strotmann (Universität Osnabrück)
Maud Szusterman (Université Paris Cité)
Christoph Thäle (Ruhr-Universität Bochum)
Marius Tiba (University of Oxford)
Vanessa Trapp (Technische Universität Hamburg)
Tara Trauthwein (University of Luxembourg)
Hiroshi Tsuji (Osaka University)
Thomas van der Jagt (Delft University of Technology)
Peter van Hintum (University of Oxford)
Katherina von Dichter (Brandenburgische Technische Universität Cottbus-Senftenberg)
Elisabeth Werner (Case Western Reserve University)
Dongmeng Xi (Shanghai University)
Joseph Yukich (Lehigh University)
Ning Zhang (Huazhong University of Science and Technology)
Chuanming Zong (Tianjin University)

Scientific Committee

Christian Buchta (Universität Salzburg)
Monika Ludwig (Technische Universität Wien)
Matthias Reitzner (Universität Osnabrück)

Local Organizers

Christian Buchta
Sarah-Leigh Lederer
Bettina Sereinig

PROGRAM

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9.30 – 10.15	Plenary talk Zong	Plenary talk Werner	Plenary talk Bárány	Plenary talk Calka	Plenary talk Colesanti
Coffee break 10.15 – 10.45					
10.45 – 11.10	Talk Fodor	Talk Yukich	Talk van der Jagt	Talk Tsuji	Talk Trapp
11.10 – 11.35	Talk van Hintum / Tiba	Talk Naszódi	Talk Khezeli	Talk Xi	Talk Schulte
11.35 – 12.00	Talk Zhang	Talk Besau	Plenary talk **11.35 – 12.20** Thäle	Talk Aramyan	Plenary talk **11.35 – 12.20** Schneider
Lunch					
15.00 – 15.45	Plenary talk Gardner	Plenary talk Hug	**13.45:** Excursion	Plenary talk Gusakova	
Coffee break 15.45 – 16.15					
16.15 – 16.40	Talk Mußnig	Talk Beck		Talk Ortega-Moreno	
16.40 – 17.05	Talk Knörr	Talk Ivanov		Talk Schuster	
	Poster session with drinks and snacks			**19.00:** Concert & Reception	