STOCHASTIC GEOMETRY WORKSHOP ABSTRACTS

23-25 NOVEMBER 2022

1. Wenesday

1.1. **TBA. Abstract**

1.2. **Crofton formula in Finsler geometry.** Léo Mathis

Abstract The classical Crofton formula says that the length of a curve on the unit sphere is equal (up to a multiple) to the average number of points of intersection with a great sphere. In a joint work with M. Stecconi we prove a similar and new formula for random hypersurfaces in Finsler manifolds that I will explain here. More precisely, given a smooth manifold with a random hypersurface (satisfying some regularity assumptions) we show how to build a Finsler structure, i.e. a norm in each tangent space, such that a Crofton formula holds. The proof relies on a Kac-Rice type formula and the construction of a section of convex bodies on the manifolds and uses the recently introduced zonoid algebra.

1.3. **Intrinsic volumes in integral geometry and valuation theory.** Thomas Wannerer

Abstract In this talk, I will approach the question of how the intrinsic volumes of subsets of the sphere or hyperbolic space should be defined from a valuation's perspective. I will try to explain how Alesker's theory a valuations on manifolds leads to a natural notion of intrinsic volumes on any riemannian manifold. The intrinsic volumes thus defined are distinguished by two properties. They are invariant under (1) formation of Alesker products and (2) pullback along isometric immersions. After reviewing consequences for the integral geometry of real and complex space forms, I will discuss what happens if we consider curvature measures localizing the intrinsic volumes.

Based on joint work with Joseph Fu.

1.4. **Random tessellations.** Anna Gusakova

Abstract: A tessellation in $\mathbb{R}^d$ is a countable locally-finite collection of convex polytopes, which cover the space and have disjoint interiors. Random tessellations are among the most central objects studied in stochastic geometry. Their analysis is motivated by their rich inner-mathematical structures, but in equal measure also by the wide range of applications in which they arise. However, there are only very few mathematically tractable models for which rigorous results are available and which do not require an analysis purely by computer simulations.

In this talk we consider a few models of this kind, which lead to random simplicial tessellations, and their corresponding dual tessellations. One of the models is a classical Poisson-Delaunay tessellation, whose construction is based on the homogeneous Poisson point process. The other three models, called β-, β'- and Gaussian-Delaunay tessellations, has been introduced recently and their construction is based on a space-time paraboloid hull process and generalizes that of the classical Poisson-Delaunay tessellation. The number of results as the probabilistic description of the typical cell, central limit theorems and connection between models will be presented.

Based on the joint work with Christoph Thäle and Zakhar Kabluchko.

2. Thursday

2.1. **Asymptotics of dynamic Boolean models.** Hanna Döring

Abstract One way to model telecommunication networks are static Boolean models. However, dynamics such as node mobility have a significant impact on the performance evaluation of such networks. Consider a Boolean model in the plane and a random direction movement scheme. Take time as the third dimension, we model these movements via cylinders. Applying Stein's method, we derive central limit theorems for functionals of the union of these cylinders. The volume, the number of isolated cylinders and the Euler characteristic of the random set are considered and give an answer to the achievable throughput, the availability of nodes, and the topological structure of the network. I will also present two more cylinder models. This is joint work with Carina Betken, Stephan Bussmann and Lianne de Jonge.

2.2. **Some insights from the geometry of numbers.** Claire Burrin

Abstract I will discuss some results from the geometry of numbers and their use in studying the shapes of random lattices and discrete lattice orbits at the hand of both classical and recent results.

2.3. **Nonarchimedean Integral Geometry.** Antonio Lerario

Abstract In this seminar I will explain a recent construction, introduced in a joint work with P. Burgisser and A. Kulkarni, for studying the "Riemannian geometry" of nonarchimedean manifolds. This construction uses a nonarchimedean version of Sard's lemma and the coarea formula, and leads to a nonarchimedean version of integral geometry, with applications to Schubert calculus and fewnomial systems.

2.4. **Beta random polytopes and angles of random simplices.** Zakhar Kabluchko

Abstract Let $X_1, \ldots, X_n$ be independent random points in the d-dimensional unit ball with density proportional to $(1 - \|x\|^2)^\beta$, where $\beta > -1$ is a parameter. For $\beta = 0$ we recover the uniform distribution on the unit ball, the limiting case $\beta \to -1$ corresponds to the uniform distribution on the unit sphere, while the case $\beta \to \infty$ corresponds to the standard Gaussian distribution. The convex hull $[X_1, \ldots, X_n]$ is called the beta polytope (with parameters n, d, β). We shall review results on the expected number of k-dimensional faces of beta polytopes and two closely related classes of polytopes called beta' and the beta* polytopes. Several objects in stochastic geometry such as the typical cell of the Poisson-Voronoi tessellation or the zero cell of the homogeneous Poisson hyperplane tessellation (in Euclidean space or on the sphere) are related to beta' polytopes, while their analogues in the hyperbolic space are related to beta* polytopes. The expected face numbers of these polytopes can be computed exactly.

3. Friday

3.1. **Curvature measures and soap bubbles beyond convexity.** Daniel Hug

Abstract A fundamental result in differential geometry states that if a smooth hypersurface in a Euclidean space encloses a bounded domain and one of its mean curvature functions is constant, then it is a Euclidean sphere (Soap Bubble Theorem). Major contributions are due to Alexandrov (1958) and Korevaar–Ros (1988).

While the smoothness assumption is seemingly natural at first thought, based on the notion of curvatures measures of convex bodies Schneider (1979) established a characterization of Euclidean spheres among general convex bodies by requiring that one of the curvature measures is proportional to the

boundary measure. We describe extensions in two directions: (1) The role of the Euclidean ball is taken by a "nice" gauge body (Wulff shape) and (2) the problem is studied in a non-convex and non-smooth setting. Thus we obtain characterization results for finite unions of Wulff shapes (bubbling) within the class of mean-convex sets or even for general sets with positive reach. Several related results are established: a Steiner–Weyl type formula for arbitrary closed sets in a uniformly convex normed vector space, formulas for the derivative of the localized volume function of a compact set and general versions of the Heintze–Karcher inequality.

(Based on joint work with Mario Santilli)

3.2. **Algebraic integral geometry.** Andreas Bernig

Abstract Let (M, G) be an isotropic pair, i.e. M is a Riemannian manifold and G a subgroup of the isometry group that acts transitively on the sphere bundle of M. A special case of the kinematic formulas allows to compute the kinematic integral $\int_G \#(X \cap gY) dg$, where $X, Y \subset M$ are submanifolds of complementary dimension. If M is compact, this is proportional to $\mathbb{E}\#(g_1 X \cap g_2 Y)$, where $g_1, g_2 \in G$ are uniformly distributed.

In the euclidean case $M = \mathbb{R}^n$, the formulas were established by Chern-Blaschke-Santaló. In the hermitian case $M = \mathbb{C}^n$, the formulas were found in collaboration with J. Fu; on complex projective space $M = \mathbb{CP}^n$ in collaboration with J. Fu and G. Solanes. A few more exotic cases (related to quaternions and octonions) are also known.

I will explain in my talk how such formulas can be found using Alesker's theory of valuations on manifolds.