Abstracts

Enumerative combinatorics from an algebraic-geometric point of view

22 March, 9:00–10:10

Christos Athanasiadis

University of Athens

Much of enumerative combinatorics studies natural statistics on sets of combinatorial objects, as well as various properties of the corresponding generating polynomials. One can often gain insight by interpreting these polynomials algebraically or geometrically. This introductory lecture will describe interesting situations of this type, by providing examples and explaining some theory. We will meet Eulerian and derangement polynomials, together with Coxeter group analogues and symmetric function generalizations, on the combinatorial side, and Stanley-Reisner rings, triangulations of spheres and balls and group actions on the cohomology of posets and toric varieties, on the algebraic-geometric side.

On f-vectors and h-vectors of relative simplicial complexes

22 March, 10:30–11:05

Giulia Codenotti

Freie Universität Berlin

A relative simplicial complex is a collection of sets given as the set theoretic difference of two simplicial complexes. Relative complexes played key roles in recent advances in algebraic and geometric combinatorics, but many questions about their general combinatorial structure are unanswered. I will start from the basic definitions, such as the notion of f-vector, a vector whose entries record the number of faces of each dimension of the complex, and introduce some of the questions and theorems about simplicial complexes, such as the Kruskal-Katona theorem, that are interesting to explore in the relative world.

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22 March, 11:05–11:40

The classification of empty 4-simplices

Óscar Iglesias-Valiño

University of Cantabria

A lattice d-simplex is the convex hull of $d+1$ affinely independent integer points in R^d. It is called empty if it contains no lattice point apart of its $d+1$ vertices. The classification of empty 3-simplices is known since 1964 (White), based on the fact that they all have width one. But for dimension 4 no complete classification is known. Haase and Ziegler (2000) computed all empty 4-simplices up to determinant 1000 and based on their results conjectured that after determinant 179 all empty 4-simplices have width one or two. We prove this conjecture as follows:

- We show that no empty 4-simplex of width three or more can have determinant greater than 7000, by combining the recent classification of hollow 3-polytopes (Averkov, Krümpelmann and Weltge) with general methods from the geometry of numbers.
- We continue the computations of Haase and Ziegler up to determinant 7600, and find that no new 4-simplices of width larger than two arise. In particular, we give the whole list of empty 4-simplices of width larger than two, which is as computed by Haase and Ziegler. In the case of empty 4-simplices of width not greater than 2 we present some new families that were not classified before. This results will lead to a complete classification of empty 4-simplices with the help of some additional upper bounds for the volume of this particular simplices.

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22 March, 14:30–15:05

Representation stability on the cohomology of complements of subspace arrangements

Artur Rapp

Philipps-Universität Marburg

We study representation stability in the sense of Church and Farb of sequences of cohomology groups of complements of arrangements of linear subspaces in real and complex space as S_n-modules. We consider arrangement of linear subspaces defined by sets of diagonal equalities $x_i = x_j$ and invariant under the action of S_n permuting the coordinates. We provide bounds on the point when stabilization occurs and an alternative proof for the fact that stabilization happens. The latter is a special case of very general stabilization results proved independently by Gadish and by Petersen and for the pure braid space the result is part of the work of Church and Farb. For this space better stabilization bounds were obtained by Hersh and Reiner.

The coordinate ring of a convex polyomino

22 March, 15:05–15:40

Claudia Andrei

University of Bucharest

The coordinate ring of a convex polyomino was introduced by Qureshi in [*Ideals generated by 2-minors, collections of cells and stack polyominoes*, Journal of Algebra, 2012, 279–303]. In that paper, it was shown that for a convex polyomino $\mathcal{P}$, the attached ideal $I_{\mathcal{P}}$ is a Cohen-Macaulay prime ideal and also a classication of the Gorenstein stack polyominoes was given. We classify all convex polyomino whose coordinate rings are Gorenstein. For this, we use a representation of the coordinate ring of $\mathcal{P}$ as an edge ring of a suitable bipartite graph. In addition, we compute the Castelnuovo-Mumford regularity of the coordinate ring of any stack polyomino in terms of the smallest interval which contains its vertices.

K3 polytopes and smooth tropical quartic surfaces

22 March, 16:00–16:35

Marta Panizzut

TU Berlin

The connected components of the complement of a tropical hypersurface are called regions, and they are convex polyhedra. A smooth tropical quartic surface of genus one has exactly one bounded region. We say that a 3-dimensional polytope is a K3 polytope if it arises as the bounded region of a smooth tropical quartic surface.

In this talk I will report on the results obtained in a joint project with Gabriele Balletti while investigating combinatorial properties of K3 polytopes. In particular I will focus on the computational challenges.

Defectivity of families of full-dimensional point configurations

22 March, 16:35–17:10

Christopher Borger

Otto-von-Guericke-Universität Magdeburg

The mixed discriminant of a family of point configurations encodes (if it is non-trivial) the conditions under which an associated system of Laurent polynomials has a multiple root. It presents a generalization of the A-discriminant of a single point configuration. However, the mixed discriminant may also be trivial and in this case we call the family of point configurations defective. Using a combinatorial criterion by Furukawa and Ito we give a necessary condition for defectivity of a family of full-dimensional configurations. This implies the conjecture by Cattani, Cueto, Dickenstein, Di Rocco and Sturmfels that a family of n full-dimensional configurations in $\mathbb{Z}^n$ is defective if and only if the mixed volume of the convex hulls of its elements is 1. This is joint work with Benjamin Nill.

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23 March,
9:00–10:10

Sparse representations from moments

Bernard Mourrain

Inria Sophia Antipolis

Recovering a hidden structure from measurements, observations, evaluation, statistics etc. is a problem that is encountered in many domains such functionnal approximation, signal processing, geometric modeling, medical imaging, etc. It has a long history going back to the work of G. de Prony on the decomposition of a function as a sum of exponential functions or the work of J.J. Sylvester on the decomposition of a binary form as a sum of powers of linear forms, or even more recently Berlekamp-Massey approach for decoding algebraic codes.

One of the motivation of the presentation is to show that these different problems fall in the same framework, which applies in several dimensions. Another motivation is to show that these decomposition problems can be analyzed and solved efficiently by algebraic methods. We will see that they reduce to the problem of decomposition of series as polynomial-exponential series. We will show that polynomial-exponential series are naturally in correspondance with Artinian Gorenstein algebras. This leads to a characterization of Hankel operators of finite rank, relating the rank of the operator with inverse systems of multiple points.

By exploiting standard eigenvector methods for solving polynomial equations, we can compute the frequencies and weights of a minimal polynomial-exponential series, using truncated pieces of the Hankel operator. A key ingredient of the approach is the flat extension criteria, which leads to a multivariate generalization of a rank condition for a Carathéodory-Fejèr decomposition of multivariate Hankel matrices. We will also describe an algorithm to compute a border basis of the Artinian Gorenstein algebra, based on a Gram-Schmidt orthogonalization process.

The approach will be illustrated in different multivariate decomposition problems: convolution operator of finite rank, reconstruction of measures as weighted sums of Dirac measures, representation of multivariate polynomial-exponential functions, sparse interpolation of polylog functions, tensor decomposition. Some connections with polynomial optimisation, semi-definite programming, compressed sensing and super-resolution shall also be discussed, if there is a moment left.

D-optimal saturated designs for the Bradley-Terry paired comparison model

23 March, 10:30–11:05

Frank Röttger

Otto-von-Guericke-Universität Magdeburg

Optimal design theory for nonlinear regression studies local optimality on a given design space. We identify the Bradley-Terry paired comparison model with graph representations and prove for an arbitrary number of parameters, that every saturated D-optimal design is displayed as a path in the graph representation. Via this path property we give a complete description of the optimality regions of saturated designs. Furthermore, we exemplify the unsaturated D-optimal designs with full support for 4 parameters.

Mass-action networks with the isolation property

23 March, 11:05–11:40

Alexandru Iosif

Otto-von-Guericke-Universität Magdeburg

Mass-action networks with the isolation property are chemical reaction networks with nice algebraic and combinatorial properties. We prove that the Zariski closure of the positive steady state variety of a mass-action network with the isolation property is a toric variety.

This is joint work with Carsten Conradi and Thomas Kahle.

On the Betti numbers of Gorenstein monomial curves

23 March, 14:30–15:05

Esra Emine Zengin

Balıkesir University

The minimal free resolution of a finitely generated k-algebra is a very important source to extract information about the algebra. Therefore, finding an explicit minimal free resolution of a standard k-algebra is one of the main problems in commutative algebra. Motivated by the fact that very little is known about the Hilbert function of a Cohen-Macaulay local ring, in this talk, our main aim is to give the minimal free resolution of the tangent cone of non-complete intersection Gorenstein monomial curves in affine 4-space that the minimal number of generators of their tangent cone is five. We show that the possible Betti sequences are $(1, 5, 6, 2)$ and $(1, 5, 5, 1)$ and compute the Hilbert function of the tangent cone of these families of curves as a result.

This is a joint work with Pınar Mete.

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23 March, 15:05–15:40

Strong persistence property for monomial ideals

Jonathan Toledo

Cinvestav IPN

An ideal I has the strong persistence property if $(I^{k+1} : I) = I^k$ for $k \geq 1$. In this talk we study this property for some families of ideals. In particular, we prove that the monomial ideals whose minimal set of generators has degree two have the strong persistence property. We show that a simple hypergraph has the strong persistence property if and only if at least one of its connected components has the strong persistence property. Also, we prove that an unmixed König simple hypergraph without 4-cycles has the strong persistence property. Furthermore, we show that a monomial ideal I has the strong persistence property if and only if its weighted ideal I_w has the strong persistence property. We prove the last result for the persistence property. Finally, we introduce and study the symbolic strong persistence.

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23 March, 16:00–16:35

The trace minimal resolution

Erika Ordog

Duke University

It has been an open problem since the 1960s to construct minimal free resolutions of arbitrary monomial ideals. In this talk, I will explain how to do that in characteristic 0, without making arbitrary choices. The differential is constructed using Moore-Penrose pseudoinverses of differentials of simplicial complexes related to the monomial ideal. In special cases, the differential "averages" geometrically or combinatorially defined resolutions. For example, in three variables, the pseudoinverse is the average of all differential quasi-inverses that come from spanning trees of the upper Koszul complex, and in this way, the trace minimal resolution is an average of the minimal free resolutions of the ideal by planar graphs. I will explain how to determine the resulting minimal free resolution in three variables by looking at the staircase surface. The methods are connected to higher-dimensional spanning trees and matrix-tree theorems in topological combinatorics.

Translative group actions on simplicial posets

23 March, 16:35–17:10

Alessio D'Alì

Max Planck Institute for Mathematics in the Sciences, Leipzig

Simplicial posets are generalizations of (face posets of) simplicial complexes. Informally, we can think of them as being obtained by gluing simplices together by any subcomplex of their boundaries. Building on previous work by Yuzvinsky and Stanley, we study how to attach a ring to these (not necessarily finite) structures and we investigate the effect of certain group actions on our construction. This has applications in the theory of abelian arrangements. This is ongoing work with Emanuele Delucchi.

Lower Bounds for Polynomials via Circuit Polynomials

24 March, 9:30–10:05

Henning Seidler

TU Berlin

Finding the minimum of a multivariate real polynomial is a well-known hard problem with various applications. We present an implementation to approximate such lower bounds via sums of non-negative circuit polynomials (SONCs). We provide a test-suite, where we compare our approach, using different solvers, with several solvers for sums of squares (SOS), including Sostools and Gloptipoly It turns out that the circuit polynomials yield bounds competitive to SOS in several cases, but using much less time and memory.

Representation Theory and Fast Matrix Multiplication

24 March, 10:05–10:40

Tim Seynnaeve

Max Planck Institute for Mathematics in the Sciences Leipzig

Determining the algorithmic complexity of matrix multiplication is one of the central open problems in computer science. This problem can be shown to be equivalent to determining the Waring rank of the *symmetrized matrix multiplication tensor* $SM_n \in S^3(\mathfrak{sl}_n^*)$. Motivated by this, we study the plethysm $S^k(\mathfrak{sl}_n)$ of the adjoint representation $\mathfrak{sl}_n$ of the Lie group SL_n. This talk is based on a current work in progress with Mateusz Michalek.

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24 March, 11:00–11:35

The Relative Canonical Ideal

Kostas Karagiannis

Aristotle University of Thessaloniki

The *canonical ideal* of a complete, non singular curve C of genus $g \geq 3$ over an algebraically closed field k is the kernel I of its *canonical embedding* $C \hookrightarrow \mathbb{P}_k^{g-1}$, defined by Ω, the sheaf of regular differentials on C. A classical result by Max Noether, Enriques and Petri ensures that, in most cases, I is generated by its elements of degree 2. In joint work with Hara Charalambous and Aristides Kontogeorgis, we find explicit generators for one class of Artin-Schreier curves over k using elements from Gröbner theory and discrete geometry. Following work of Kontogeorgis-Karanikolopoulos and Bertin-Mézard we obtain the curve's lift to characteristic 0, a Kummer curve, and apply a similar method to get its canonical ideal. Finally, we use deformation-theoretic arguments to obtain the *relative* canonical ideal, i.e. the canonical ideal of the family of curves whose special and generic fibres are the Artin-Schreier and Kummer curves respectively.

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24 March, 11:35–12:10

Isotropic and Coisotropic Subvarieties of Grassmannians

Kathlén Kohn

TU Berlin

Every projective variety is uniquely defined by a polynomial in the projectivized coordinate ring of some Grassmannian. This polynomial has the same degree as the given projective variety and is known as its Chow form. Gel'fand, Kapranov and Zelevinsky introduce coisotropic forms as generalizations of Chow forms in their famous book. We generalize their results even further: we do not only consider hypersurfaces in Grassmannians, but also subvarieties with larger codimension. Moreover, we introduce a dual notion of coisotropy, called isotropic varieties. This leads us to a full description of the classically studied classification of congruences, i.e. surfaces in the Grassmannian of lines in projective 3-space.
