BOUNDS FOR THE NON-VANISHING OF THE LOCAL COHOMOLOGY
AND HOMOLOGY MODULES

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Let $M$ be a module over a Noetherian local ring $(R, \mathfrak{m}, k)$ and denote its $i^{th}$ local cohomology with respect to $\mathfrak{m}$ by $H^i(M)$. When $M$ is finitely generated a Grothendieck result asserts that

$$
sup\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} = \dim M \text{ and } \inf\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} = \text{depth}(M).$$

For an arbitrary module $M$ we still have the lower bound, at least if we replace the classical depth by the Ext-depth with respect to $\mathfrak{m}$. For the upper bound we only have

$$
sup\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} \leq \dim R/\text{Ann}_R(M),$$

that is because $H^i_{\mathfrak{m}}(M) = H^i(\check{\mathcal{C}}^R_M)$, where $\check{\mathcal{C}}_x^R$ denotes the Čech complex built on a parameter system of $R/\text{Ann}_R(M)$. In the present talk we will sharpen this bound.

More generally we also consider the local cohomology and homology modules of an unbounded complex with respect to some ideal of a commutative ring. These are well defined and we will provide some bounds for their non-vanishing. Our results hold not only for ideals of a Noetherian ring, but also for finitely generated ideals of a commutative ring, generated by a weakly pro-regular sequence. The Ext-depth and the Tor-codepth will play a rôle.

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