BOUNDS FOR THE NON-VANISHING OF THE LOCAL COHOMOLOGY AND HOMOLOGY MODULES

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Let M be a module over a Noetherian local ring $(R, \mathfrak{m}, \Bbbk)$ and denote its i^{th} local cohomology with respect to $\mathfrak{m}$ by $H^i(M)$. When M is finitely generated a Grothendieck result asserts that

$$\sup\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} = \dim M \text{ and}$$
$$\inf\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} = \operatorname{depth}(M).$$

For an arbitrary module M we still have the lower bound, at least if we replace the classical depth by the Ext-depth with respect to $\mathfrak{m}$. For the upper bound we only have

$$\sup\{i \mid H^i_{\mathfrak{m}}(M) \neq 0\} \leq \dim R/\operatorname{Ann}_R(M),$$

that is because $H^i_{\mathfrak{m}}(M) = H^i(\check{C}_{\underline{x}} \otimes_R M)$, where $\check{C}_{\underline{x}}$ denotes the Čech complex built on a parameter system of $R/\operatorname{Ann}_R(M)$. In the present talk we will sharpen this bound.

More generally we also consider the local cohomology and homology modules of an unbounded complex with respect to some ideal of a commutative ring. These are well defined and we will provide some bounds for their non-vanishing. Our results hold not only for ideals of a Noetherian ring, but also for finitely generated ideals of a commutative ring, generated by a weakly pro-regular sequence. The Ext-depth and the Tor-codepth will play a rôle.

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