TRACE FOR ENDOMORPHISMS ON TRACE MODULES

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The module M over the commutative ring A is called a trace module, when $M = \tau(x)M$ for all $x \in M$, where $\tau(x)$ is the ideal $\{\mu(x) \mid \mu \in M^*\}$. These modules are also called, locally projective, universally torsionless and by Raynaud and Gruson flat strict Mittag-Leffler modules. Every projective module is a trace module and every trace module is flat. When M is a trace module the natural map $M^* \otimes_A M \to \mathrm{End}_A(M)$ is injective and its image consists of all endomorphisms f, such that $\mathrm{Im}\, f$ is contained in some finitely generated submodule of M. For such a map f corresponding to $z \in M^* \otimes_A M$, I define its trace as $\mathrm{Tr}(f)$ as $e(z)$, where $e : M^* \otimes_A M \to A$ is the evaluation map. This generalises the ordinary trace for an endomorphism on a finitely generated projective module. The alternating powers of a trace module turn out also to be trace modules. This I use to define the characteristic polynomial of f as the polynomial(!)
$\sum_0^\infty Tr(\wedge^i(f))$.

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